

Hong Kong Mathematics Olympiad (2006 – 2007)

Heat Event (Individual)

香港数学竞赛 (2006 – 2007)

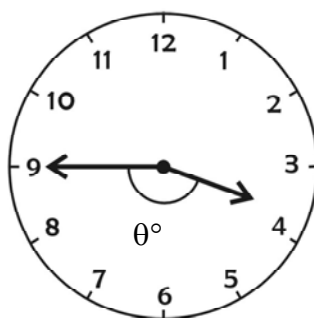
初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 如图一，时钟显示着三时四十五分。若时针与分针的交角是 θ° ，求 θ 的值。

In Figure 1, a clock indicates the time 3:45. If the angle between the hour-hand and the minute-hand is θ° , find the value of θ .

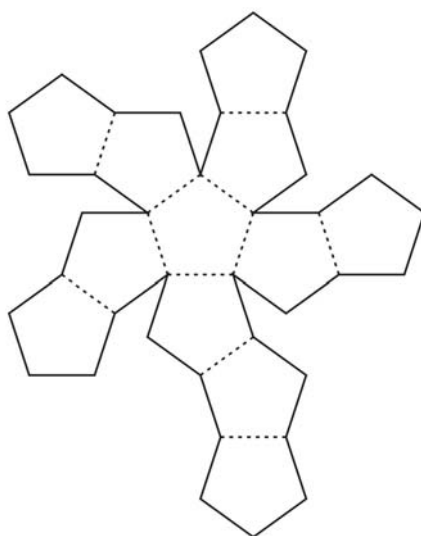


图一

Figure 1

2. 如图二的折纸图样能折出一个正多面体。若该正多面体有 y 条棱，求 y 的值。

In Figure 2, there is a paper net that can be wrapped into a regular polyhedron. If this polyhedron has y edges, find the value of y .



图二

Figure 2

3. 在 4 本英文书、6 本中文书及 9 本日文书中任取两本。已知这两本书是相同语言的。若有 X 个不同的选择，求 X 的值。

Among 4 English books, 6 Chinese books and 9 Japanese books, two books are selected. It is found that they are of the same language. If there are X such choices, find the value of X .

4. 设 r_1 和 r_2 是方程 $(x-2006)(x-2007)=2007$ 的两个实根。若 r 是方程 $(x-r_1)(x-r_2)=-2007$ 较小的实根，求 r 的值。

Let r_1 and r_2 be the two real roots of the equation $(x-2006)(x-2007)=2007$. If r is the smaller real root of the equation $(x-r_1)(x-r_2)=-2007$, find the value of r .

5. 已知 α 及 β 是方程 $x^2-5^{2007}x+5^{1000}=0$ 的根。若 $s=\log_{25}\frac{\alpha^2}{\beta}+\log_{25}\frac{\beta^2}{\alpha}$ ，求 s 的值。

Given that α and β are the roots of the equation $x^2-5^{2007}x+5^{1000}=0$. If $s=\log_{25}\frac{\alpha^2}{\beta}+\log_{25}\frac{\beta^2}{\alpha}$, find the value of s .

6. 对任意实数 a 、 b 、 c 及 d ，定义运算 $*$ ：

$$(a, b) * (c, d) = (ad + bc, bd).$$

若 $(x, y) = \left(1, \frac{3}{7-\sqrt{5}}\right) * (8+\sqrt{5}, 3)$ 及 $a = \frac{x}{y}$ ，求 a 的值。

For any real number a , b , c and d , we define the operation $*$:

$$(a, b) * (c, d) = (ad + bc, bd).$$

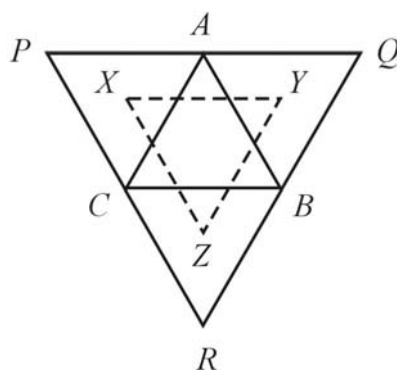
If $(x, y) = \left(1, \frac{3}{7-\sqrt{5}}\right) * (8+\sqrt{5}, 3)$ and $a = \frac{x}{y}$, find the value of a .

7. 已知 $\sin \alpha - \cos \alpha = \frac{1}{5}$ 及 $0^\circ < \alpha < 180^\circ$ 。若 $\tan \alpha = B$ ，求 B 的值。

Given that $\sin \alpha - \cos \alpha = \frac{1}{5}$ and $0^\circ < \alpha < 180^\circ$. If $\tan \alpha = B$, find the value of B .

8. 如图三， $\triangle PAC$ 、 $\triangle QBA$ 、 $\triangle RCB$ 及 $\triangle ABC$ 皆是等边三角形。点 X 、 Y 及 Z 分别为 $\triangle PAC$ 、 $\triangle QBA$ 及 $\triangle RCB$ 的内心。若 PA 的长度是 10 cm 及 $\triangle XYZ$ 的周界是 w cm，求 w 的值。(注：三角形的内心为该三角形三条内角平分线的交点。)

In Figure 3, $\triangle PAC$, $\triangle QBA$, $\triangle RCB$ and $\triangle ABC$ are equilateral triangles. The points X , Y and Z are the incentre of $\triangle PAC$, $\triangle QBA$ and $\triangle RCB$ respectively. If the length of PA is 10 cm and the perimeter of $\triangle XYZ$ is w cm, find the value of w . (Remark: the incentre of a triangle is the point of intersection of the three interior angle bisectors of the triangle.)



图三

Figure 3

9. 设 $f(x) = \frac{1}{2}(4x^2 - 60x + 9 + |4x^2 - 60x + 9|)$ ，若 $k = f(1) + f(2) + f(3) + \cdots + f(15) + f(16)$ ，求 k 的值。

Let $f(x) = \frac{1}{2}(4x^2 - 60x + 9 + |4x^2 - 60x + 9|)$. If $k = f(1) + f(2) + f(3) + \cdots + f(15) + f(16)$, find the value of k .

10. 在平面上点 P 的坐标是 $(-3, 4)$ 。以 $(0, 0)$ 为中心，点 P 顺时针方向旋转 45° 后，再沿 y -轴反射到达点 $Q = (x, y)$ 。若 $z = x + y$ ，求 z 的值。

The coordinates of point P on the plane is $(-3, 4)$. After rotating 45° clockwise about the centre $(0, 0)$ and reflecting along the y -axis, the point P reaches the point $Q = (x, y)$. If $z = x + y$, find the value of z .